mental data shows that, with the hand initially extended in back, the thrower exerts a moderate positive (forward) force. As the hand accelerates forward, the force increases to a maximum and then decreases to zero. When the throwing hand is extended in front, the applied force becomes negative. The positive force is the result of muscular contractions, and the strong negative force at the completion of the throw is mainly the result of ligaments in the joints resisting further extension. Wrist torque is also assumed to be the result of muscular contractions. The mass and moment of inertia of the throwing hand are obtained from their physical dimensions plus the assumption that the hand density is that of water.

The idea of modeling the act of throwing by a few physical parameters, hand mass and hand radius of gyration plus force and wrist torque, as measurable functions of hand position was inspired by the work of Paul Klopsteg on the physics of bows and arrows.[6] He characterized the operation of a bow by two parameters; the available energy stored in the bow limbs and the virtual mass of the bow limbs. The virtual mass is derived from data on arrow kinetic energy versus arrow mass. In his simple model the fraction of the available energy stored in the bow limbs that is imparted to an arrow is a function only of the arrow mass: the efficiency = arrow mass/(arrow mass + virtual mass). For a given bow, the concepts of available energy and virtual mass of the bow make it easy to calculate the velocity for arrows of any mass.

Cotterell and Kamminga applied Klopsteg's concept of virtual mass to hand thrown and atlatl thrown spears.[7] They used a much more detailed model for throwing, which included the mass and moment of inertia of the forearm and hand. They also assumed that the angular velocities of the elbow and wrist joints are fixed and independent of the masses being moved. This independence is in contrast with the model presented here, which assumes a predetermined force and torque.

The thrower simultaneously applies a force and wrist torque. The horizontal component of force causes horizontal acceleration. It also causes angular acceleration when the force is not collinear with the center of mass. Additional angular acceleration comes from the wrist torque and the vertical component of force.

Wrist torque is also important with a hand-thrown projectile. A baseball pitcher, throwing off the tips of his fingers, applies wrist rotation over a lever arm extending from his wrist joint to his fingertips, approximately 16 to 20 cm. With a hand-thrown spear, the wrist rotation is applied to a lever, wrist joint to palm, which is only about 10 cm long. The use of an atlatl allows the optimization of the lever length for the dart mass and physical limitations of the thrower. The model treats hand thrown and atlatl thrown darts as essentially the same, and only the length and mass of the lever arm are different. The model shown in Fig. 1 assumes that the throwing hand has a rigid grip on the atlatl so that the hand and atlatl can be considered as one rigid body. Newton's laws of motion are used to compute the relation between applied effort and dart velocity.

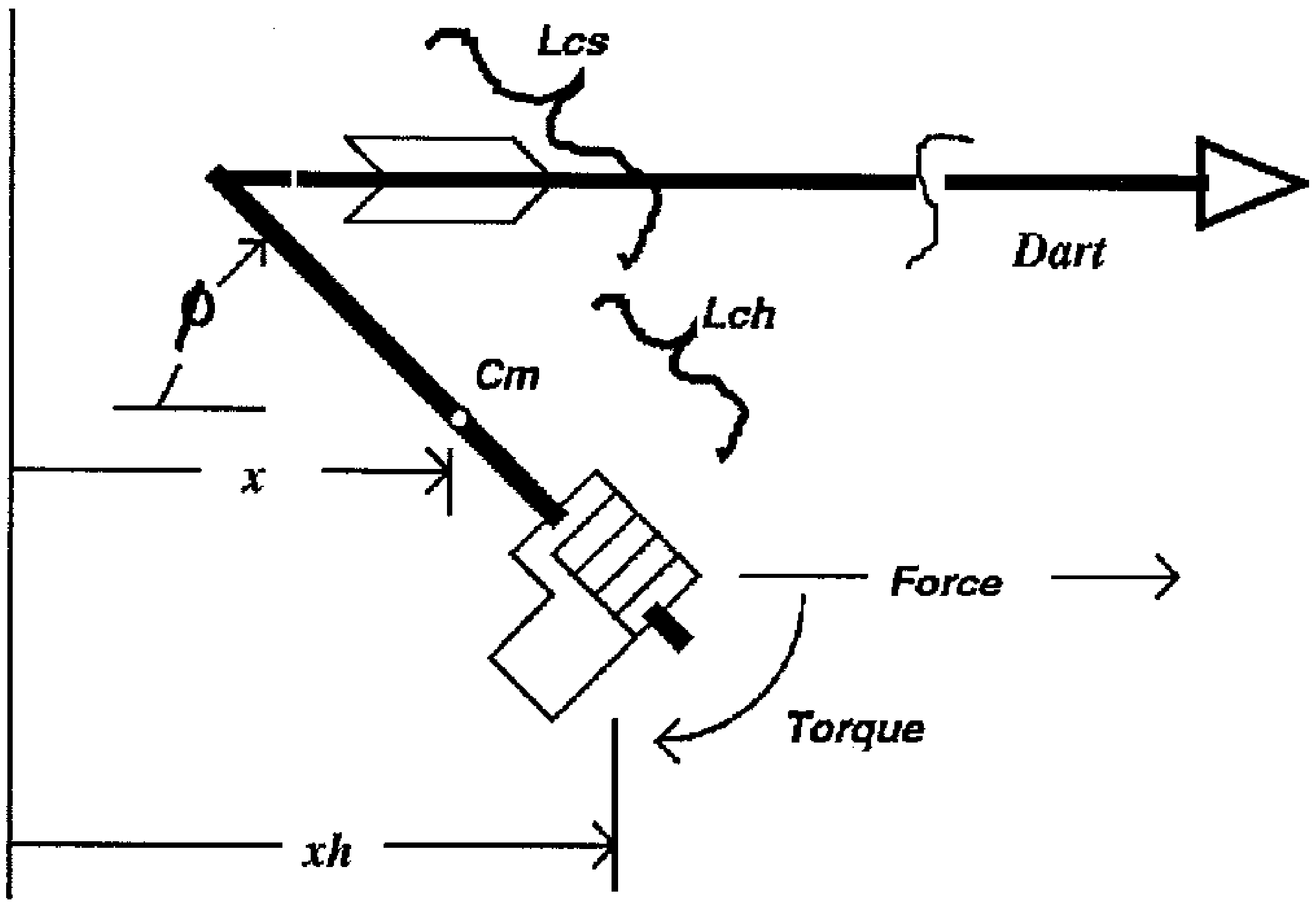

Fig. 1. The geometry of an atlatl throw. The quantities x and x_h are the horizontal positions of the center of mass and hand.

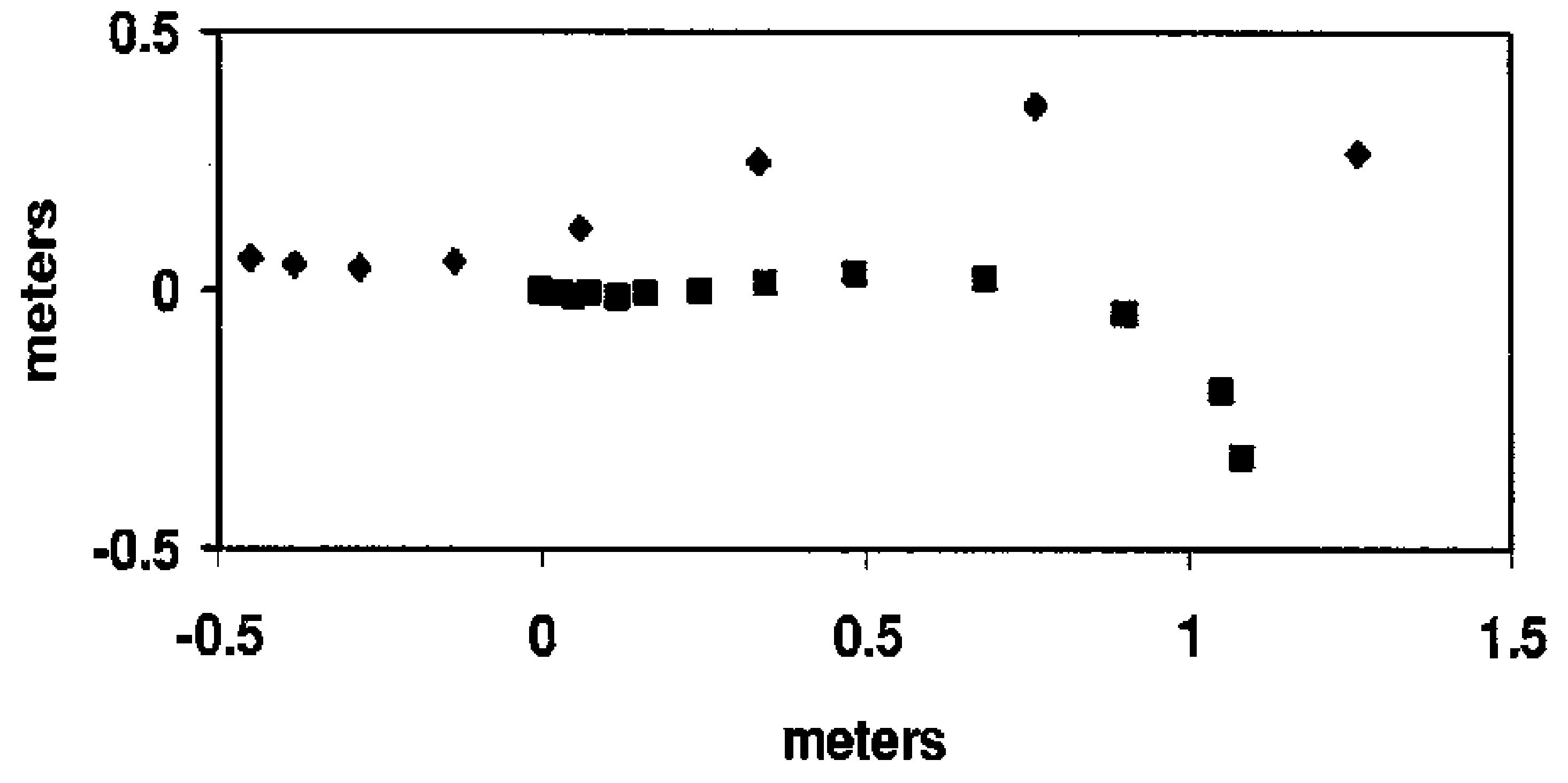

Fig. 2. Atlatl position versus time with 0.02 s time intervals.

IV. THE EXPERIMENT

Position versus time data was obtained with a high-speed video digitizer. Small tabs of reflecting tape were placed on the atlatl near the hand and the spur and on the dart at the middle, rear, and front ends to reflect light into the digitizer. Several throws were made to verify that the equipment was working correctly. In the final throw the horizontal and vertical positions of all five spots were measured every $\frac{5}{1000}$ s and recorded for later computer analysis. Figure 2 shows the atlatl position every 0.02 s. The horizontal and vertical scales are both in meters. The initial position is on the left, the final on the right.

Table I summarizes the physical dimensions of the atlatl, dart, and equivalent mass, and the radius of gyration of the hand used in the experiment. From the measured positions versus the time and the masses and dimensions of the equipment, we can calculate the force and torque applied by the thrower. The thrower, an athletic 50 year old man, launched the dart with a "moderate" effort, the sort of throw that one would use for accuracy rather than trying for maximum distance. The following assumptions were made to analyze the data: (a) The horizontal velocity of the dart is the quantity of interest. (b) The hand and atlatl are considered to be one

Table I. Physical parameters for the experimental atlatl and dart.

L_a, atlatl length, wrist (proximal) to spur (distal)	0.61 m
M_a, atlatl mass (uniformly distributed)	135 g
Added weight, M_w, at a distance $L_w = 0.324$ m from the wrist	78 g
M_d, dart mass	71 g
M_h, hand mass	680 g
R_h, hand radius of gyration	0.051 m
I_c, moment of inertia	0.022 kg-m^2